MATH 1650: SECTION 6.2: LOGARITHMIC FUNCTIONS WORKSHEET

Exponential functions $f(x) = b^x$ are are one-to-one which means they are invertible. In this section, we explore their inverses, the *logarithmic functions* which are called 'logs' for short. In other words...

$b^a = c$ if and only if $\log_b(c) = a$. That is, $\log_b(c)$ is the exponent you put on b to obtain c.

EXAMPLE: Find the exact value of the logarithms below:

- $\log_3(81)$ is the exponent you put on _____ to get _____ .

  Hence, $\log_3(81) =$ _____.

- $\log_2\left(\frac{1}{8}\right)$ is the exponent you put on _____ to get _____ .

  Hence, $\log_2\left(\frac{1}{8}\right) =$ _____.

- $\log_{25}(5)$ is the exponent you put on _____ to get _____ .

  Hence, $\log_{25}(5) =$ _____.

- **NOTE:** '$\log_{10}(x)$' is usually written as '$\log(x)$.'

  $\log(1000)$ is the exponent you put on _____ to get _____ .

  Hence, $\log(1000) =$ _____.

- **NOTE:** '$\log_e(x)$' is usually written as '$\ln(x)$.'

  $\ln(\sqrt{e^3})$ is the exponent you put on _____ to get _____ .

  Hence, $\ln(\sqrt{e^3}) =$ _____.

- $\log_2(2^{1.3})$ is the exponent you put on _____ to get _____ .

  Hence, $\log_2(2^{1.3}) =$ _____.

  NOTE: In general, $\log_b(b^x) = x$ for all real numbers. Do you see why?

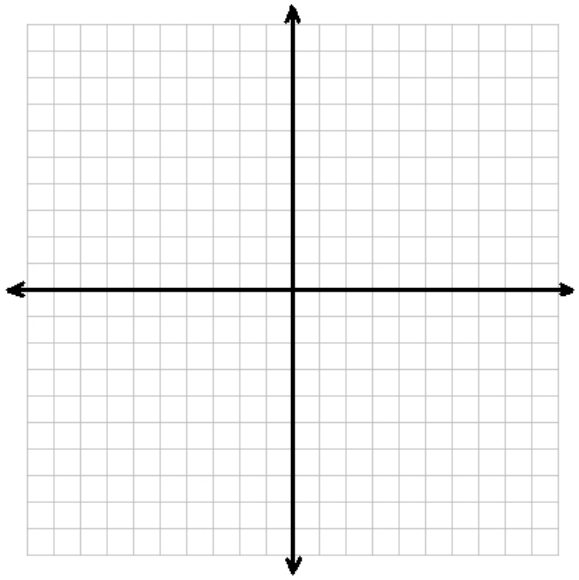
- $\log_3(5)$ is the exponent you put on _____ to get _____ .

  Hence, $3^{\log_3(5)} =$ _____.

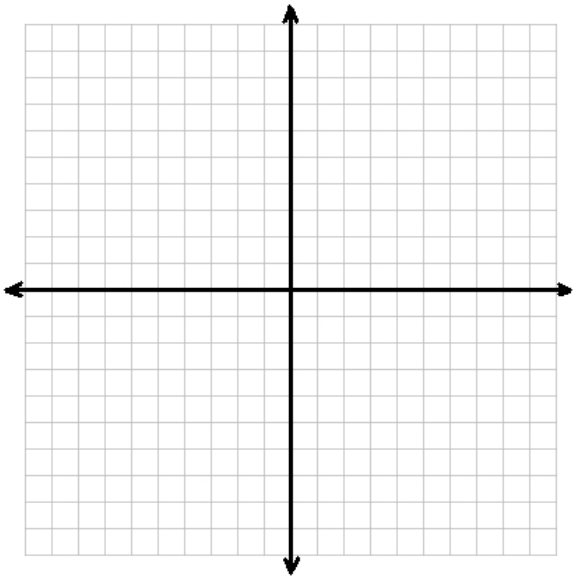
  NOTE: In general, $b^{\log_b(x)} = x$ for all $x > 0$. Do you see why?

EXAMPLE:

- Graph $y = \log_2(x)$ by starting with $y = 2^x$.
  Label three points on each graph and the asymptotes.

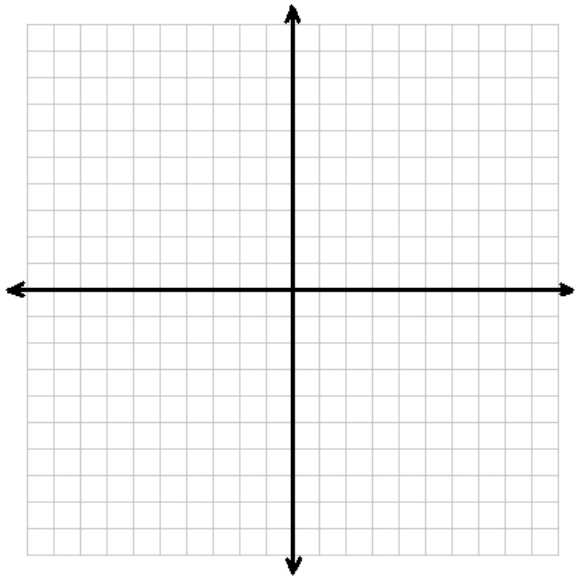


$y = 2^x$

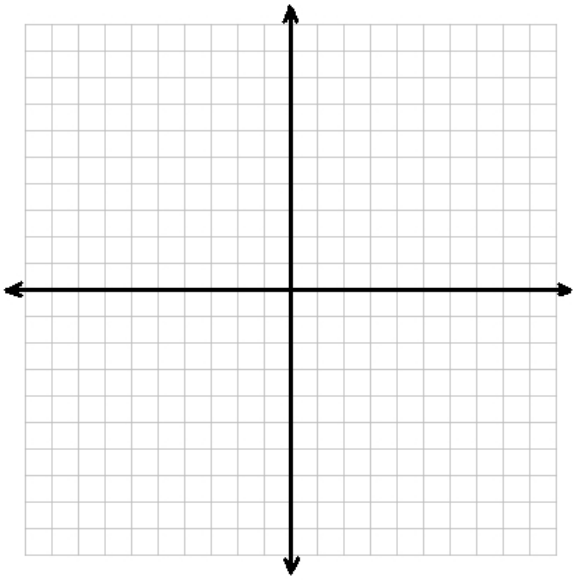


$y = \log_2(x)$

- Graph $f(x) = 3\log_2(x+1)$ by starting with $y = \log_2(x)$ and using transformations.
  Label three points on each graph and the asymptotes.



$y = \log_2(x)$



$f(x) = 3\log_2(x+1)$

EXAMPLE:

- Explain why $\log_2(-1)$ is not a real number.

- Explain why $\log_2(0)$ is not a real number.

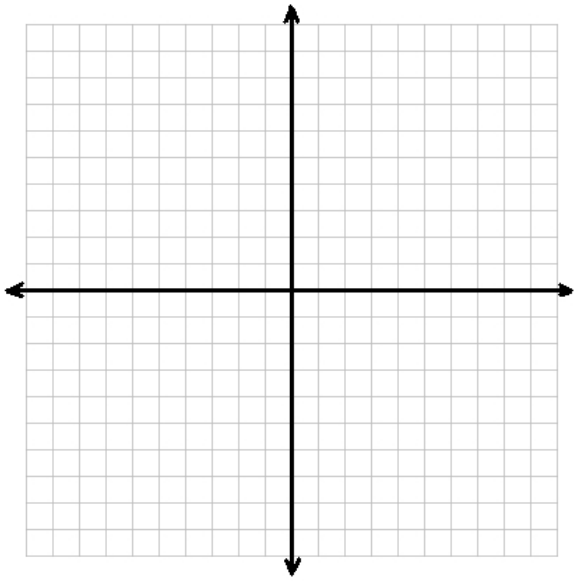
EXAMPLE: Find the domain of the following functions algebraically and check your answer graphically. Write your answers using interval notation.

- $f(x) = 2\log(3 - x) - 1$

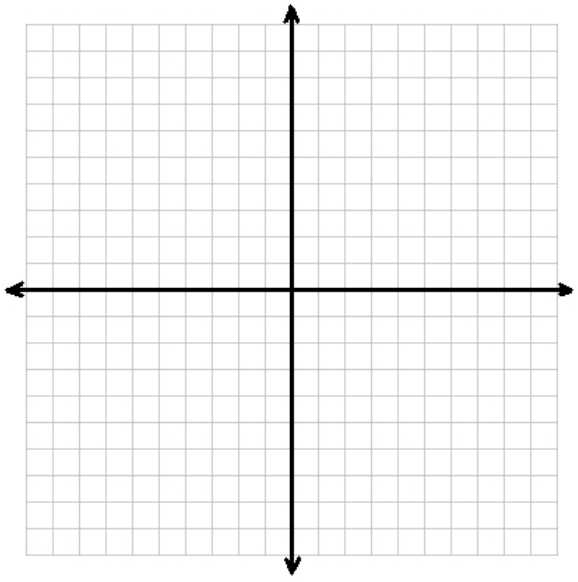
- $g(x) = \ln\left(\frac{x}{x-1}\right)$

EXAMPLE: Let $f(x) = 2^{x-1} - 3$.

- Graph f. State the domain and range of f.



- Explain why f is invertible and graph f^{-1}. State the domain and range of f^{-1}.



EXAMPLE: (Continued.) Let $f(x) = 2^{x-1} - 3$.

- Find a formula for $f^{-1}(x)$.

  HINT: Think procedurally... and use the fact that logarithms undo exponentials.

- Verify $\left(f^{-1} \circ f\right)(x) = x$ and $\left(f \circ f^{-1}\right)(x) = x$ for all x in the appropriate domains.

- Use f^{-1} to solve: $2^{x-1} - 3 = 4$

HOMEWORK: Section 6.2: 1 - 81 every other odd.